2016年6月5日 13:25

1. A: $\vec{n_1} = (4, 3, -1)$

$\vec{n_2} = (2, -1, 1)$

$$\vec{l} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} i & j & k \\ 4 & 3 & -1 \\ 2 & -1 & 1 \end{vmatrix} = (2, -6, -10)$$

$z = 0$

$$\begin{cases} 4x + 3y = 0 \\ 2x - y + 5 = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{3}{2} \\ y = 2 \end{cases}$$

$\Rightarrow$ equation for intersection line:

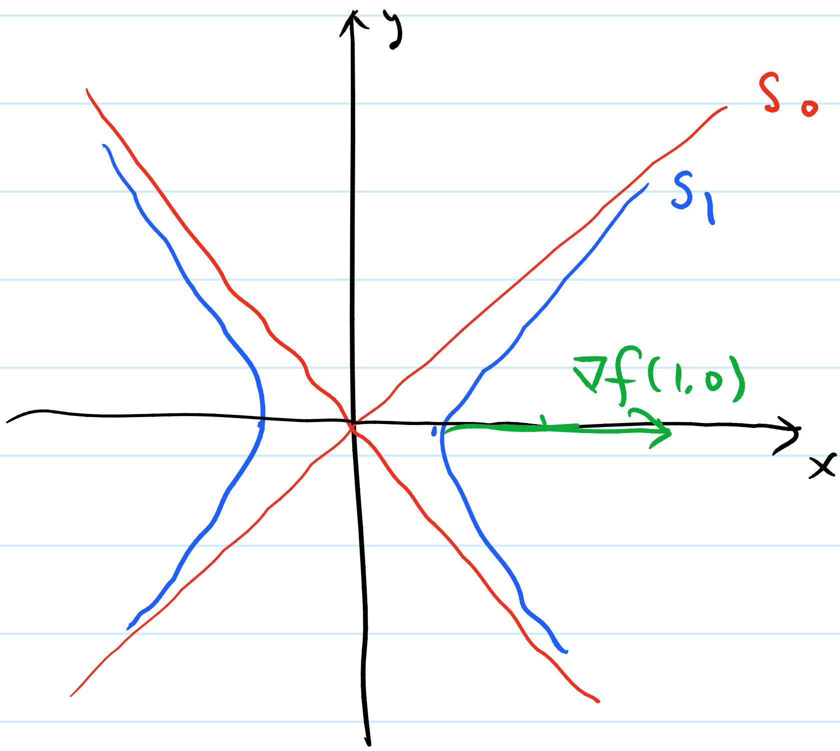
$$\begin{cases} x = -\frac{3}{2} + t \\ y = 2 - 3t \\ z = -5t \end{cases}$$

B: $$\cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|} = \frac{2\sqrt{39}}{39}$$

2. $S_0 = x^2 - y^2 = 0$

$S_1 = x^2 - y^2 = 1$

$\nabla f(1,0) = (2x, -2y)\big|_{(1,0)} = (2,0)$



key point: S_1 approaches S_0 as $x \to \pm\infty$

3. A: $\vec{v} = \dfrac{d\vec{r}}{dt} = (-e^{-t}, -6\sin 3t, 6\cos 3t)$

$\vec{v}(0) = (-1, 0, 6)$ velocity

$|\vec{v}| = \sqrt{37}$ speed

B: $\vec{r}(0) = (1, 2, 0)$

$\Rightarrow$ equation for tangent line

$$\begin{cases} x = 1 - t \\ y = 2 \\ z = 6t \end{cases}$$

4. (A) cf: tutorial notes.

(B) $f'(x) = 4x - 3$ $\quad f'(0) = -3$

$f''(x) = 4$

$\Rightarrow \kappa(0) = \dfrac{|4|}{[1 + (-3)^2]^{\frac{3}{2}}} = \dfrac{\sqrt{10}}{25}$

5. $\vec{T} = \dfrac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|} = \frac{1}{5}(3\cos t, -3\sin t, 4)$

$\dfrac{d\vec{T}}{dt} = \frac{1}{5}(-3\sin t, -3\cos t, 0)$

$N = \dfrac{\frac{d\vec{T}}{dt}}{\left|\frac{d\vec{T}}{dt}\right|} = (-\sin t, -\cos t, 0)$

$B = \vec{T} \times \vec{N} = \begin{vmatrix} i & j & k \\ \frac{3}{5}\cos t & -\frac{3}{5}\sin t & \frac{4}{5} \\ -\sin t & -\cos t & 0 \end{vmatrix}$

$= (\frac{4}{5}\cos t, -\frac{4}{5}\sin t, -\frac{3}{5})$

$\vec{a} = \dfrac{d^2\vec{r}}{dt^2} = (-3\sin t, -3\cos t, 0)$

$\vec{a} = a_T\vec{T} + a_N\vec{N}$

$\Rightarrow a_T = \vec{a}\cdot\vec{T} = 0$

$a_N = \vec{a}\cdot\vec{N} = 3$

6. A: $\frac{\partial f}{\partial x}\Big|_{(1,0)} = (8x + e^x - y^2)\Big|_{(0,1)} = 0$

B $\frac{\partial f}{\partial y}\Big|_{(0,1)} = (14y - 2xy)\Big|_{(0,1)}$

$= 14$

C: $f(0,1) = 1 + 7 + 3 = 11$

$F(x,y,z) = f(x,y) - z = 0$

$\Rightarrow$ tangent plane at $(0, 1, f(0,1))$

$0 \cdot (x-0) + 14 \cdot (y-1) - (z-11) = 0$

i.e. $14y - z - 3 = 0$

7. $\frac{dy}{dx}\Big|_{(-1,1)} = -\frac{F_x}{F_y}\Big|_{(-1,1)}$

$= -\frac{y-3}{x+2y}\Big|_{(-1,1)}$

$= 2$

8. A: $\frac{\partial w}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r}$

$$= f_x \cos\theta + f_y \sin\theta$$

$$\frac{\partial w}{\partial \theta} = f_x \cdot (-r\sin\theta) + f_y \cdot r\cos\theta$$

B: $(f_r)^2 + \frac{1}{r^2}(f_\theta)^2$

$= (f_x)^2 + (f_y)^2$ (direct calculation)